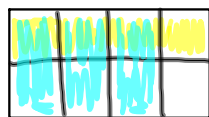


6.3 Completed Notes

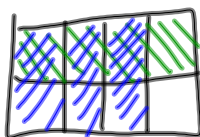
6.3: Multiplication and Division of Rational Numbers

Definition: If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$.

Example: Draw a figure to represent $\frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$.



$\frac{3}{4}$



intersection of shading is numerator
number of squares is denominator

Example: Calculate $\frac{27}{62} \cdot \frac{8}{54}$.

$$\frac{27}{62} \cdot \frac{8}{54} = \frac{2}{31}$$

(Handwritten: 27 and 8 are crossed out, leaving 31 and 4. 4 and 54 are crossed out, leaving 2 and 13.5. The final result is 2/31.)

$$\frac{27}{62} \cdot \frac{8}{54} = \frac{2}{31}$$

(Handwritten: 27 and 8 are crossed out, leaving 31 and 4. 4 and 54 are crossed out, leaving 2 and 13.5. The final result is 2/31.)

Example: Calculate $\frac{18}{44} \cdot \frac{55}{27}$.

$$\frac{18}{44} \cdot \frac{55}{27} = \frac{5}{6}$$

(Handwritten: 18 and 55 are crossed out, leaving 9 and 11. 11 and 44 are crossed out, leaving 9 and 4. 9 and 27 are crossed out, leaving 1 and 3. The final result is 5/6.)

$$\frac{18}{44} \cdot \frac{55}{27} = \frac{5}{6}$$

(Handwritten: 18 and 55 are crossed out, leaving 9 and 11. 11 and 44 are crossed out, leaving 9 and 4. 9 and 27 are crossed out, leaving 1 and 3. The final result is 5/6.)

Fact: The rational numbers over multiplication have the closure, commutative, and associative properties. The following properties also hold.

Identity:

$$1 \cdot \frac{a}{b} = \frac{a}{b} \cdot 1 = \frac{a}{b}$$

Inverse:

$$\frac{a}{b} \cdot \frac{b}{a} = \frac{b}{a} \cdot \frac{a}{b} = 1 \quad \left(\frac{a}{b} \neq 0\right)$$

Zero Multiplication Property: $0 \cdot \frac{a}{b} = \frac{a}{b} \cdot 0 = 0$

Distributive: $\frac{a}{b} \cdot \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \cdot \frac{c}{d} + \frac{a}{b} \cdot \frac{e}{f}$

Example: Calculate the following.

(a) $3\frac{1}{3} \cdot 3\frac{1}{3}$

$$\begin{aligned} &= \left(3 + \frac{1}{3}\right) \left(3 + \frac{1}{3}\right) \\ &= 3 \cdot 3 + 3 \cdot \frac{1}{3} + \frac{1}{3} \cdot 3 + \frac{1}{3} \cdot \frac{1}{3} \\ &= 9 + 1 + 1 + \frac{1}{9} = 11\frac{1}{9} = \frac{100}{9} \end{aligned}$$

(b) $2\frac{2}{3} \cdot 1\frac{1}{4}$

$$2\frac{2}{3} \cdot 1\frac{1}{4} = \frac{10}{3} \cdot \frac{5}{4} = \frac{10}{3} = 3\frac{1}{3}$$

6.3 Completed Notes

Definition: If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers with $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d}$ is the unique rational number $\frac{e}{f}$ such that $\frac{c}{d} \cdot \frac{e}{f} = \frac{a}{b}$.

We will not be studying a model for this in class, but look at p. 390 for some ideas of how to teach this.

Example: Show that $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$. *because $\frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$*

$$\frac{3}{4} \times \bigcirc = \frac{2}{3}$$

$$\frac{\cancel{3}^1}{4} \times \frac{\cancel{8}^2}{\cancel{9}_3} = \frac{2}{3}$$

Example: Show that $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$.

$$\textcircled{1} \frac{4}{3} \cdot \frac{3}{4} \cdot n = \frac{2}{3} \cdot \frac{4}{3}$$

$$n = \frac{2}{3} \cdot \frac{4}{3}$$

$$\textcircled{2} \frac{\frac{2}{3}}{\frac{3}{4}} \cdot \frac{\frac{4}{3}}{\frac{4}{3}} = \frac{\frac{2}{3} \cdot \frac{4}{3}}{1} = \frac{2}{3} \cdot \frac{4}{3}$$

$$\textcircled{3} \frac{2}{3} \div \frac{3}{4} \cdot \left(\frac{\cancel{3}}{\cancel{4}} \cdot \frac{4}{3} \right) = \frac{2}{3} \cdot \frac{4}{3}$$

Theorem: If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers and $\frac{c}{d} \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \quad (\text{Keep-Change-Flip})$$

~~Proof:~~

Example: Compute $\frac{4}{5} \div \frac{12}{5}$ using Keep Change Flip with one of the explanations from before.

$$\frac{4}{5} \div \frac{12}{5} = n$$

$$\text{Then } \frac{\cancel{5}^1}{12} n = \frac{4}{5} \cdot \frac{\cancel{5}^1}{12}$$

$$n = \frac{\cancel{4}^1}{\cancel{5}_1} \cdot \frac{\cancel{8}^1}{\cancel{12}_3} = \frac{1}{3}$$

$$\frac{3}{4} \div \frac{3}{8}$$

$$= \frac{\frac{3}{4}}{\frac{3}{8}} \cdot \frac{\frac{8}{3}}{\frac{8}{3}} = \frac{\cancel{3}^1}{4} \cdot \frac{\cancel{8}^2}{\cancel{3}_1} = 2$$